Summer Homework
For 2019-2020
Math Methods

Ready for Year 2: _____

PAPER 1 – NO CALCULATOR ALLOWED

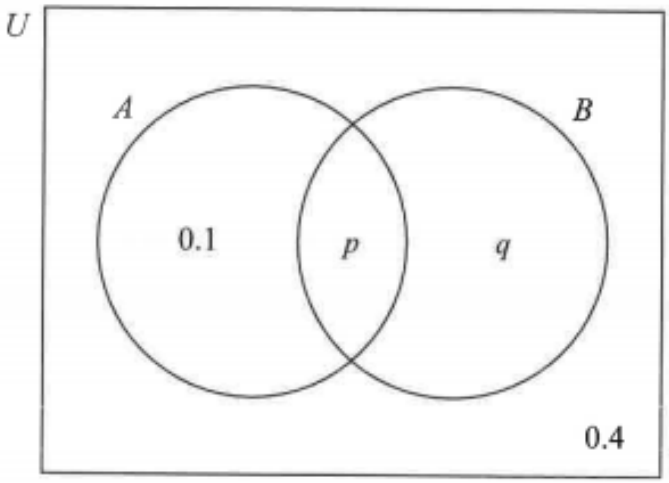
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following Venn diagram shows the events A and B, where $\mathrm{P}(A) = 0.3$. The values shown are probabilities.



(a) Find the value of p. [2]

(b) Find the value of q. [2]

(c) Find $\mathrm{P}(A' \cup B)$. [2]

(This question continues on the following page)

3. [Maximum mark: 7]

The following diagram shows a right triangle ABC. Point D lies on AB such that CD bisects $A\hat{C}B$.

diagram not to scale

$A\hat{C}D = \theta$ and $AC = 14\,\text{cm}$

(a) Given that $\sin\theta = \frac{3}{5}$, find the value of $\cos\theta$. [3]

(b) Find the value of $\cos 2\theta$. [2]

(c) Hence or otherwise, find BC. [2]

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4. [Maximum mark: 6]

Let $f(x)=\frac{2x-1}{x+3}$, $x \neq -3$.

(a) Write down the equation of the vertical asymptote of the graph of f. [1]

(b) Find $f^{-1}(x)$. [3]

(c) Find the equation of the horizontal asymptote of the graph of f^{-1}. [2]

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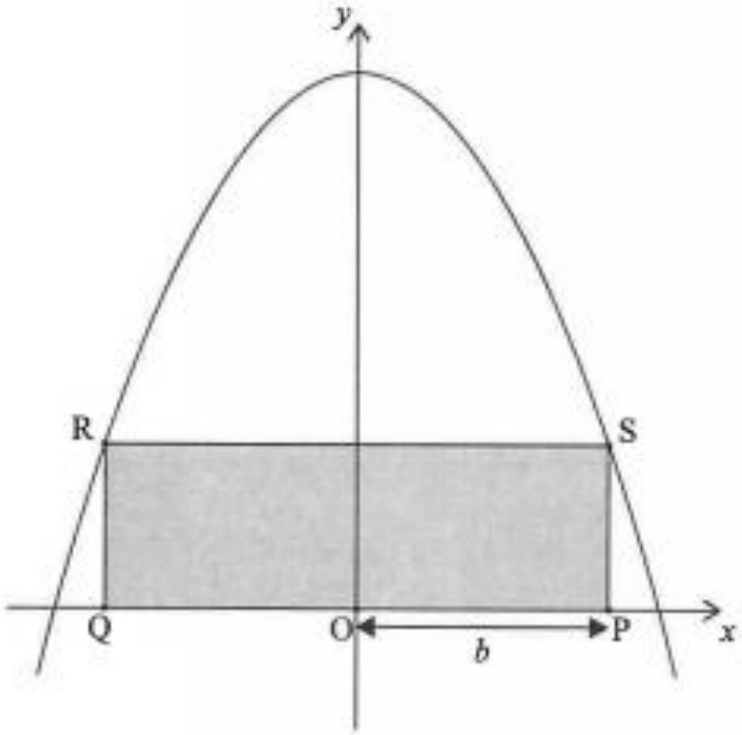
8. [Maximum mark: 16]

Let $f(x) = 9 - x^2$, $x \in \mathbb{R}$.

(a) Find the x-intercepts of the graph of f. [2]

The following diagram shows part of the graph of f.



Rectangle PQRS is drawn with P and Q on the x-axis and R and S on the graph of f.

Let $OP = b$.

(b) Show that the area of PQRS is $18b - 2b^3$. [2]

(c) Hence find the value of b such that the area of PQRS is a maximum. [5]

Consider another function $g(x) = (x-3)^2 + k$, $x \in \mathbb{R}$.

(d) Show that when the graphs of f and g intersect, $2x^2 - 6x + k = 0$. [2]

(e) Given that the graphs of f and g intersect only once, find the value of k. [5]

10. [Maximum mark: 17]

Consider $f(x) = \sqrt{x}\sin\left(\frac{\pi}{4}x\right)$ and $g(x) = \sqrt{x}$ for $x \geq 0$. The first time the graphs of f and g intersect is at $x = 0$.

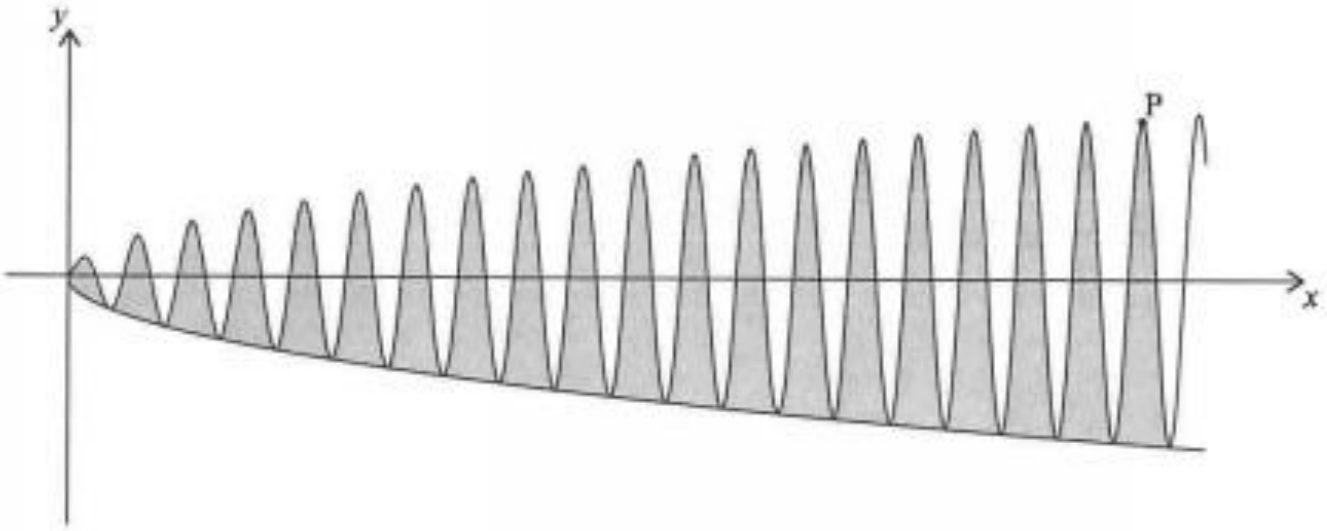
(a) Find the **two** smallest non-zero values of x for which $f(x) = g(x)$. [5]

The set of all non-zero values that satisfy $f(x) = g(x)$ can be described as an arithmetic sequence, $u_n = a + bn$ where $n \geq 1$.

(b) Find the value of a and of b. [4]

(c) At point P, the graphs of f and g intersect for the 21st time. Find the coordinates of P. [4]

The following diagram shows part of the graph of g **reflected** in the x-axis. It also shows part of the graph of f and the point P.



(d) Find an expression for the area of the shaded region. Do not calculate the value of the expression. [4]

PAPER 2 – CALCULUATOR ALLOWED

2. [Maximum mark: 6]

Consider the graph of the function $f(x) = a(x + 10)^2 + 15$, $x \in \mathbb{R}$.

(a) Write down the coordinates of the vertex. [2]

(b) The graph of f has a y-intercept at -20. Find a. [2]

(c) Point $P(8, b)$ lies on the graph of f. Find b. [2]

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5. [Maximum mark: 6]

A jigsaw puzzle consists of many differently shaped pieces that fit together to form a picture.

Jill is doing a 1000-piece jigsaw puzzle. She started by sorting the edge pieces from the interior pieces. Six times she stopped and counted how many of each type she had found. The following table indicates this information.

Edge pieces (x)	16	31	39	55	84	115
Interior pieces (y)	89	239	297	402	580	802

Jill models the relationship between these variables using the regression equation $y = ax + b$.

(a) Write down the value of a and of b. [3]

(b) Use the model to predict how many edge pieces she had found when she had sorted a **total** of 750 pieces. [3]

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6. [Maximum mark: 7]

Consider the expansion of $\left(x^2+1.2\right)^n$ where $n \in \mathbb{Z}$, $n \geq 3$. Given that the coefficient of the term containing x^6 is greater than $200\,000$, find the smallest possible value of n.

7. [Maximum mark: 7]

The first terms of an infinite geometric sequence, u_n, are $2, 6, 18, 54, \ldots$.
The first terms of a second infinite geometric sequence, v_n, are $2, -6, 18, -54, \ldots$.

The terms of a third sequence, w_n, are defined as $w_n = u_n + v_n$.

(a) Write down the first three **non-zero** terms of w_n. [3]

The finite series, $\sum_{k=1}^{225} w_k$, can also be written in the form $\sum_{k=0}^{m} 4r^k$.

(b) Find the value of

(i) r;

(ii) m. [4]

10. [Maximum mark: 16]

There are three fair six-sided dice. Each die has two green faces, two yellow faces and two red faces.

All three dice are rolled.

(a) (i) Find the probability of rolling exactly one red face.

(ii) Find the probability of rolling two or more red faces. [5]

Ted plays a game using these dice. The rules are:

- Having a turn means to roll all three dice.
- He wins \$10 for each green face rolled and adds this to his winnings.
- After a turn Ted can either:
  - end the game (and keep his winnings), or
  - have another turn (and try to increase his winnings).
- If two or more red faces are rolled in a turn, all winnings are lost and the game ends.

(b) Show that, after a turn, the probability that Ted adds exactly \$10 to his winnings is $\frac{1}{3}$. [5]

The random variable D (\$) represents how much is added to his winnings after a turn.

The following table shows the distribution for D, where \$$w$ represents his winnings in the game so far.

D (\$)	$-w$	0	10	20	30
$\mathrm{P}(D=d)$	x	y	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{27}$

(c) (i) Write down the value of x.

(ii) Hence, find the value of y. [3]

Ted will always have another turn if he expects an increase to his winnings.

(d) Find the least value of w for which Ted should end the game instead of having another turn. [3]